Math Circle Lesson: Candy-Sharing
Shannon Rogers

Abstract
If we give a bunch of children several pieces of candy, and force them to share with their neighbors in a particular way (with no eating or cheating!), what happens?
Let's explore this and other related sharing scenarios.

Supplies
Bag of Jolly Ranchers or similarly wrapped candies; something that could be used as "coal" in second scenario (different kind of candy? Suggestions. *James*: perhaps some kind of prank candy--https://www.amazon.com/Crick-ettes-Seasoned-Crickets-Pack-24/dp/B000IEZINW)

Starting Scenario
5 children sit in a circle, and they have 5 pieces of candy between them. At the start of each minute, any child who has more than 1 piece of candy gives 1 piece to each of the two children sitting next to him/her. (This happens simultaneously if more than one child has more than 1 piece of candy.) This repeats at the start of each new minute.
If they ever get to a point where every child has exactly 1 piece of candy, they win and get to eat the candy!

1) What questions can you come up with?
2) What might you change about the conditions of the problem to create new avenues of exploration?

Secondary Scenario (if we have time, a little harder to address the followup)
5 children sit in a circle. Some of the children have piece(s) of candy, and some have lump(s) of coal. (Some may have neither and be empty-handed.) No child has both candy and coal. At the start, there are more pieces of candy than lumps of coal. At the start of each minute, a child who has coal is chosen. His/her coal is magically converted to candy, but *each* of the two children next to him/her must pay a "tax" of the same number of pieces of candy as the original child had lumps of coal. If they don't have enough candy to pay the tax, they receive coal to make up the deficit that they owe.
If all the coal is eventually eliminated, then all the children get to eat their candy!

Discussion points
- Type up scenarios on a piece of paper to be handed out to participants, and then act out a few times to make sure there's understanding at the beginning of each.
- Notice I never posed a problem here, just set up a scenario. As mathematicians, we don't always know the question, let alone the answer. Teachers may get into groups to explore.
    - Questions to guide if they get stuck: Does every game stop? Are some infinite? What if we have more/fewer candies? What if we change the rules, and kids only

pass candy to one side (right or left). What if they only have to give away half their n candies (or half of n-1, if n odd); or some other fraction?

- Notations for organizing. Symmetry. Organizing your work.
- Pedagogy: We aren't covering grade-level content...so what's the point of doing something silly like this? Would something like this be useful to your students? What worked well about how this problem was presented, and how might you adapt it if you were working in your classroom or with a different population?

References

- Grant Cairns, "Equitable Candy Sharing," The American Mathematical Monthly, June-July 2017, p. 518-526
- David Patrick, "Candy Sharing," San Diego Math Teachers' Circle, October 7, 2017
- Glenn Iba and James Tanton, "Candy Sharing," The American Mathematical Monthly, January 2002, p 25-35
- http://www.themathcircle.org/researchproblems.php