

Candy Sharing Spreadsheet Activity: Instructor's Notes

Introduction

In this activity, students construct a spreadsheet that performs several turns of the candy sharing game, starting from an initial distribution of candy.

This outline assumes that the instructor and students have already explored the hands-on candy sharing activity (as outlined in “Candy Sharing Game: Lesson Plan” and “Candy Sharing Game Rules / Candy Sharing Game Challenges”), and is suitable for students who already have some experience with spreadsheets, or where the time for the activity is sufficient to include key general concepts and techniques of spreadsheets. Specifically, students must be able to write formulas in spreadsheet cells, using combinations of relative and absolute addresses, and copy and paste those formulas to other cells in the spreadsheet.

None of the formulas or operations used here are unique to any given spreadsheet product. Thus, virtually any relatively up-to-date spreadsheet software package can be used, including Microsoft Excel, Google Sheets, Apple Numbers, LibreOffice Calc, and OpenOffice Calc. Thus, this activity is generally well-suited to most computing environments, as long as some spreadsheet package is either installed locally or accessible online.

Setting up the initial table

Candy sharing takes place over 2 dimensions: across *players*, which we can number from 1 to the total number of players, and through *turns* (or time), which we can number from 0 (the initial distribution) to whatever limit we’re interested in. Thus, it makes sense to start the spreadsheet by setting up a table with these dimensions. For example, we could have the table shown in Figure 1, with 6 players and 15 turns (plus the initial distribution).

Of course, the dimensions could easily be interchanged in a spreadsheet, or even spread across multiple sheets/tabs. Similarly, while it makes sense to encourage students to select a realistic size for their spreadsheets (e.g. it’s probably best to build a spreadsheet for 10 players before doing so for 100 players), there’s a wide range of reasonable values for number of players, and for number of turns.

	A	B	C	D	E	F	G
1	Players						
2	Turns	1	2	3	4	5	6
3	0						
4	1						
5	2						
6	3						
7	4						
8	5						
9	6						
10	7						
11	8						
12	9						
13	10						
14	11						
15	12						
16	13						
17	14						
18	15						

Figure 1: Initial candy sharing table for 6 players, 15 turns.

Writing the sharing formulas

Since turn 0 represents the initial distribution of candy, that row of the table will presumably be filled in manually. For subsequent turns, however, the amount of candy for each player can be computed with a straightforward formula. Conceptually, we have

A player's candy at the end of turn $(n + 1)$ =

The same player's candy at the end of turn n

– 2 (if the player had at least 2 pieces at the end of turn n)

+ 1 (if the player to the left had at least 2 pieces at the end of turn n)

+ 1 (if the player to the right had at least 2 pieces at the end of turn n).

In the table shown in Figure 1, cell **C4** represents player 2's candy at the end of turn 1; **C3** represents player 2's candy at the end of turn 0 (i.e. the initial distribution); **B3** and **D3** represent the players immediately to the right and left of player 2 at the end of turn 0. Thus, we could translate the conceptual formula above to this Excel formula in **D4**:

$$= C3 - IF(C3 >= 2, 2, 0) + IF(B3 >= 2, 1, 0) + IF(D3 >= 2, 1, 0)$$

(Note that horizontal whitespace has been added for readability. Some spreadsheet programs accept and ignore the extra spaces; some accept them when typed, but remove them

automatically. Unfortunately, some programs will report an error on any unexpected spaces; the formula will need to be adjusted accordingly in those cases.)

Since we don't yet have data for turn 0, this formula won't show us much of interest yet.

Nonetheless, we can type the formula in **C4**, and hit the *Enter* key to store it there.

We've chosen to start with player 2, turn 1, for a reason: this formula can be copied to most of the other players without any adjustment to the cell references. In fact, we can take the next step now, by copying the formula and pasting it into the turn 1 cells for players 3 through 5, giving us the table in Figure 2. (Some students may want to paste the formula across the entire row of players at this point; that's fine, as long as the cell reference adjustments described below are then made.)

	A	B	C	D	E	F	G	
1								
2	Turns		1	2	3	4	5	6
3	0							
4	1		0	0	0	0		
5	2							
6	3							
7	4							
8	5							
9	6							
10	7							
11	8							
12	9							
13	10							
14	11							
15	12							
16	13							
17	14							
18	15							

Figure 2: Candy sharing table, after copying simple formula.

(Note that all of the cell references used so far use relative addressing – i.e. without the \$ prefix for the row numbers or column letters – so the formulas in cells **D4** through **F4** adjusted automatically on pasting, to refer to the appropriate cells in row 3.)

Now we encounter a slight complication: If we copy the same formula to cell **B4**, it will refer to cell **A3** for the player to the left's candy at the end of turn 0, but that column doesn't represent a player; instead, the player to the left in that case is actually player 6, in column **G**. So we'll need to adjust that reference.

We can copy the formula in cell **C4** and paste it to **B4**. Then, edit the formula in **B4**, replacing the reference to **A3** with a reference to **G3**:

$$= B3 - IF(B3 \geq 2, 2, 0) + IF(G3 \geq 2, 1, 0) + IF(C3 \geq 2, 1, 0)$$

Similarly, we can copy the formula in cell **F4** and paste it to cell **G4**; then edit the formula in **G4**, replacing the references to **H3** with a reference to **B3**:

$$= G3 - IF(G3 \geq 2, 2, 0) + IF(F3 \geq 2, 1, 0) + IF(B3 \geq 2, 1, 0)$$

This results in the table shown in Figure 3

	A	B	C	D	E	F	G
1							
2	Turns	1	2	3	4	5	6
3	0						
4	1	0	0	0	0	0	0
5	2						
6	3						
7	4						
8	5						
9	6						
10	7						
11	8						
12	9						
13	10						
14	11						
15	12						
16	13						
17	14						
18	15						

Figure 3: Candy sharing table complete for turn 1.

(Of course, the details of the above formulas assume that the table is located and sized as shown in the accompanying figures. Otherwise, the references will need to be adjusted accordingly.)

If we fill in the row for turn 0 with initial values (at least some of which are non-zero), we should see the turn 1 values update automatically (see example in Figure 4).

At this point, we suggest discussing with the students possible ways to check the formulas, to make sure they are correct, before copying and pasting them to the subsequent rows. For example, when multiple cells are selected, most spreadsheets will compute and display (often in one of the toolbars at the edge of the screen) the sum of the selected cells; this can be used as a quick (if incomplete) check of the formulas.

	A	B	C	D	E	F	G
1	Players						
2	Turns	1	2	3	4	5	6
3	0	2	2	0	3	3	1
4	1	1	1	2	2	2	3
5	2						
6	3						
7	4						
8	5						
9	6						
10	7						
11	8						
12	9						
13	10						
14	11						
15	12						
16	13						
17	14						
18	15						

Figure 4: Initial candy distribution in turn 0 and sharing formulas in turn 1.

When we’re reasonably confident that the formulas for turn 1 are correct, we can simply copy and paste them through the remaining turns. We can do this a number of ways. For example, we can select all the cells for one turn (e.g. cells **B4** through **G4** of Figure 4), copy, then select all of the cells in the remaining turns (e.g. cells **B5** through **G18**) and paste. Most spreadsheets also allow us to select a range that has formulas at one side or end (e.g. **B4** through **G18**) and use a “Fill Down” feature to copy those formulas to all of the cells in the range. (Unfortunately, this feature is found on different toolbars and menus for different products – and even different releases of the same product.) There are also features for selecting a range with formulas (or values), then extend that range by dragging, with the contents automatically propagating as the range is extended. We suggest encouraging your students to explore different ways of doing this operation. (See Figure 5 for an example of sharing formulas copied to 15 turns.)

	A	B	C	D	E	F	G
1	Players						
2	Turns	1	2	3	4	5	6
3	0	2	2	0	3	3	1
4	1	1	1	2	2	2	3
5	2	2	2	1	2	2	2
6	3	2	1	3	1	2	2
7	4	1	3	1	3	1	2
8	5	3	1	3	1	3	0
9	6	1	3	1	3	1	2
10	7	3	1	3	1	3	0
11	8	1	3	1	3	1	2
12	9	3	1	3	1	3	0
13	10	1	3	1	3	1	2
14	11	3	1	3	1	3	0
15	12	1	3	1	3	1	2
16	13	3	1	3	1	3	0
17	14	1	3	1	3	1	2
18	15	3	1	3	1	3	0

Figure 5: Candy sharing formulas copied for 15 turns.

Creating a graph

At this point, your students may recognize some patterns in the data – e.g. cycles or fixed points, where the distribution of candy either doesn't change at all after a given turn, or changes in such a way that a small number of distributions is repeated indefinitely. One aid in spotting such patterns is a *time series* plot, where some quantity (or several quantities describing the state of a system) is charted over time.

Spreadsheets make it easy to create time series plots, with some caveats:

- The default chart type, in many cases, isn't well-suited to a time-series plot. We recommend using an **XY (scatter)** chart type (in Google Sheets and Apple Numbers, the **Line** and **2D Line** are the appropriate types, respectively), with connecting lines between plotted points. Since candy sharing data varies by whole numbers, dot symbols can usually be omitted, with just the lines plotted.
- Depending on the number of players and the number of turns, the spreadsheet program may not recognize whether the data series are formed by the columns or the rows. For candy sharing, each player is a data series; for the example we've been using here, the series are thus formed by the columns.
- If the data selected for charting includes the player numbers or turn numbers (or both – which we do suggest), the corresponding checkboxes should be checked in the chart creation options, so that a meaningless time series of turn numbers (for example) isn't plotted along with the candy sharing data.

For example, if we select the range **A2:G18** in Figure 5, and create an **XY (scatter)** chart in LibreOffice Calc, specifying lines only, with data series in columns, and labels in the first row and first column, we will get a chart something like Figure 6. If we plot the same data in Google Sheets, using a **Line** chart, Figure 7 is the result.

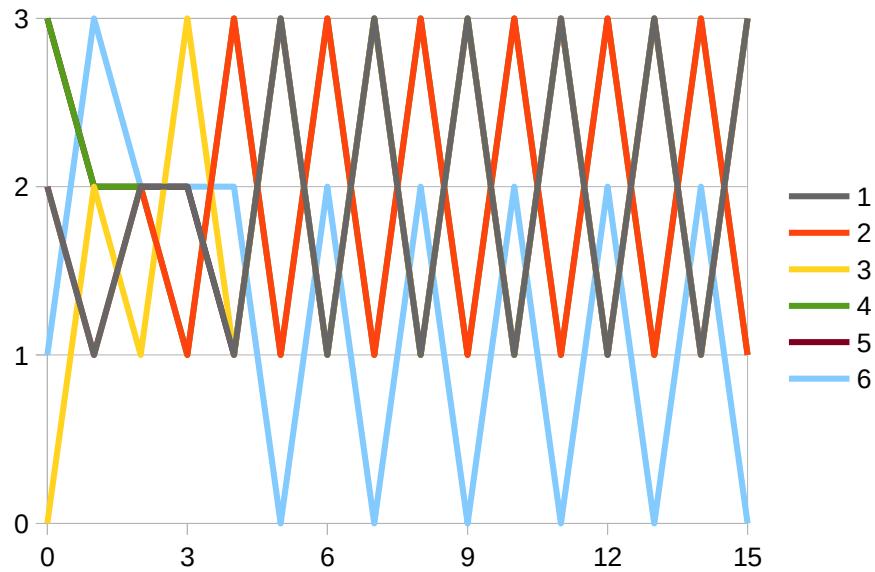


Figure 6: Candy sharing time series in LibreOffice Calc.

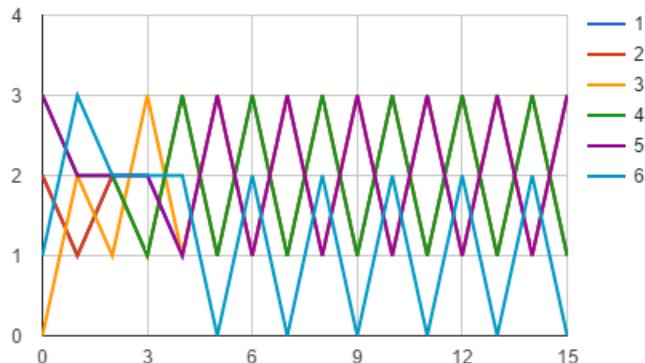


Figure 7: Candy sharing time series in Google Sheets.

Some students may notice that not all of the players' time series lines are visible over the entire time line. We suggest encouraging the students to explore the possible reasons for this, and to verify that the plot lines match the data. In this case, some of the players' candy amounts are identical to those of other players after a given turn, so some players' time series plot lines will overwrite others.

This may be an opportune moment for a discussion of effective data displays. As part of that, you might want to encourage the students to refine their charts – e.g. adding titles to the axes, the legend, or the chart itself, adjusting the colors of the plotted lines to better distinguish them, and adjusting the scales of the axes.

Math circle activity challenges and questions

The spreadsheet created so far can already be a powerful tool in answering (at least in part) the questions posed in the candy sharing math circle activity challenges. Students can experiment with different total amounts and distributions of candy to find critical thresholds where the outcome of the game fundamentally changes, and to design initial distributions that give the specified results.

Different number of players

As we work with the candy sharing table and the sharing formulas – copying and pasting them for an entire game, gaining confidence in how they work – we might want to modify them to add or remove players.

The multi-sheet capability of spreadsheet programs makes it easy to setup different tables for different game configurations – for example, with different numbers of players. Of course, a student may simplify modify the table they've already created, to add or remove players. On the other hand, copying the entire sheet (tab) containing the table to a new sheet in the same file makes it easy to create a new game configuration, without affecting the previous one.

However the students choose to add or remove players, we suggest that you encourage them to re-verify the sharing formulas, to make sure they're still valid for the new number of players. Also, it's entirely possible that if the number of players is increased, more turns will be needed to identify cyclic patterns in the candy distribution over time; thus, students may need to extend the tables and copy the sharing formulas to additional turns.

Finally, virtually any time series plot (or other chart) will need to be modified when the dimensions of the underlying data change. In some cases, because the features for changing the underlying data series of a chart are deeply nested in the chart properties, it may be easiest to delete and re-create a chart when the size of the data table changes.

Extensions

Beyond the questions posed in the math circle activity challenges handout, new questions can be raised and explored using the spreadsheet. In some case, this can be done without modifying the formulas; in other cases, such modifications can be made to explore more extensive changes to the game, such as:

- Changing the sharing threshold (i.e. instead of sharing when a player has 2 pieces of candy, a different value could be used).
- Taking individual turns sharing, instead of sharing simultaneously. That is, player 1 would share or not, based on how much candy they have; then, player 2 will share or not – but now, the amount of candy is not just based on how much player 2 had at the end of the previous turn, but also on any additional candy just received from player 1; etc.