

Candy Sharing Spreadsheet Activity: Student Handout

Introduction

In the Candy Sharing math circle activity, you explored some of the different ways that candy (or poker chips, tokens, etc.) can be distributed around a circle of players, and how different total amounts and different distributions can lead to different outcomes in the game.

In this activity, we'll continue investigating candy sharing, using a spreadsheet program (such as Microsoft Excel or Google Sheets) to increase our ability to explore alternative configurations and outcomes.

Setting up the initial table

In any given turn of the game, the amount of candy in a player's pile is based on the amount in the previous turn, and the amounts in the adjacent players' piles in the previous turn. Beyond this relationship from one turn to the next, you've already seen that the amount of candy in front of any player can change a lot through the turns of the game. Even if the game reaches a repeating pattern of candy distribution, the amounts can change from turn to turn as part of that pattern. So it probably makes sense to use the spreadsheet to keep track of all players, over several turns of the game. We can do this easily in a spreadsheet, with a rectangular table of cells.

- From your earlier exploration, how many turns do you think you might want to include in the spreadsheet? In other words, how many turns might it take for the game to end up in a fixed or repeating pattern?
- Would the answer to the above be affected by the number of players?

One possible table is shown in Figure 1. There, we have 6 players and 15 turns (plus turn 0, the initial distribution).

Of course, your table could have a different size; it might also have the rows and columns interchanged (that is, players numbered along the left side, and turns numbered along the top). Feel free to experiment with different layouts and formatting, to find an arrangement that seems clear to you.

	A	B	C	D	E	F	G
1		Players					
2	Turns	1	2	3	4	5	6
3	0						
4	1						
5	2						
6	3						
7	4						
8	5						
9	6						
10	7						
11	8						
12	9						
13	10						
14	11						
15	12						
16	13						
17	14						
18	15						

Figure 1: Initial candy sharing table for 6 players, 15 turns.

Writing the sharing formulas for a single turn

Since turn 0 represents the initial distribution of candy, you’ll probably fill that row in manually. (As you go through this activity, you might have some ideas for distributing the candy using formulas; that can be a very interesting area of investigation.) For subsequent turns, the amount of candy for each player can be computed with a straightforward formula. Conceptually, we have

A player’s candy at the end of some turn =

The same player’s candy at the end of the previous turn

– 2 (if the player had at least 2 pieces to share at the end of the previous turn)

+ 1 (if the player to the left had at least 2 pieces at the end of the previous turn)

+ 1 (if the player to the right had at least 2 pieces at the end of the previous turn).

In the table shown in Figure 1, cell **C4** represents player 2’s candy at the end of turn 1. In that case, we can see that **C3** represents player 2’s candy at the end of turn 0 (i.e. the initial distribution), while **B3** and **D3** represent the players immediately to the right and left of player 2 at the end of turn 0. That gives us almost all we need for a formula to compute player 2’s candy at the end of turn 1.

Another important piece, which you may have already seen in other spreadsheets, is the **IF** function. This function tells the spreadsheet to check some condition, and give one value if the condition is true, and another value if the condition is false. This function has the form

IF(condition, value if condition is true, value if condition is false)

(The italicized text indicates placeholders – that is, something that you would replace when you use the function in your own formulas.)

Now we can express the relationship between cells **C4**, **B3**, **C3**, and **D3** in Figure 1 with this formula in cell **C4**:

= C3 - IF(C3 >= 2, 2, 0) + IF(B3 >= 2, 1, 0) + IF(D3 >= 2, 1, 0)

Notes:

- The equals sign (=) is part of the formula. Spreadsheet formulas must begin with the equals sign, to tell the spreadsheet program that we’re writing a formula, and not just text.
- In the formula, **B3**, **C3**, and **D3** are cell references. They refer to the cells at those addresses, and when the value of the formula is computed, the values of those referred-to cells are used in the computation. (Remember that in Figure 1, those are the cells with player 1’s, player 2’s, and player 3’s candy at the end of turn 0.)
- In spreadsheet formulas, as well as most programming languages, “at least”, or “greater than or equal to”, is written with the greater than sign and the equals sign together, with no space between them: **>=**
- In this example, spaces have been added – after the equals sign, before and after the plus and minus signs, and after the commas – for readability. Some spreadsheet programs accept and ignore the extra spaces; some accept them when typed, but remove them automatically. Unfortunately, some programs will report an error on any unexpected spaces; the formula will need to be adjusted accordingly in those cases.)
- If your table is not in the same cells as the table shown in Figure 1, you’ll need to adjust the cell references: instead of **B3**, **C3**, and **D3**, you should use references to the cells for player 1, 2, and 3’s candy at the end of turn 0 in your table.

If you haven’t already put data in for turn 0, this formula won’t show us much of interest yet. Nonetheless, go ahead and type the formula in **C4**, and hit the *Enter* key to store it there.

This example starts with player 2, turn 1, for a reason: this formula can be copied to most of the other players without any adjustment to the cell references. In fact, we can take that step now:

- Select cell **C4** (that is, the cell for turn 1, player 2).

- Copy the cell to the clipboard, using the **Edit/Copy** menu or toolbar command, the *Ctrl-C* keyboard shortcut on Windows, or the **⌘-C** shortcut on OS X.
- Select cells **D4** through **F4** (that is, the cells for turn 1, players 3 through 5).
- Paste the clipboard contents into the selected cells, using the **Edit/Paste** menu or toolbar command, the *Ctrl-V* keyboard shortcut on Windows, or the **⌘-V** shortcut on OS X.

Take a moment to look at the formula for turn 1, player 3 (in **D4** in this example), and notice that in copying and pasting the formula one cell over, the cell references have shifted over as well: instead of referring to **B3**, **C3**, and **D3**, this formula refers to **C3**, **D3**, and **E3**. This automatic adjustment is one of the useful features of most spreadsheet programs. (Are there any examples you can think of where you wouldn't want the cell references to adjust automatically? In fact, there are ways to write formulas so that the references don't adjust in this fashion.)

Now, try copying and pasting from cell **C4** to cells **B4** and **G4** – that is, to the cells for the first and last players (players 1 and 6) in turn 1.

- Does the automatic adjustment of references work as expected?
- In this example, **B4** is the cell for turn 1, player 1. Who is the player to the left of player 1? (In the game, players sit in a circle; in the spreadsheet, we have them arranged in a row – or possibly in a column – but sharing is still done in a logical circle. So who sits to the left of player 1? Who sits to the right of player 6?)
- Does the formula in **B4** correctly refer to the turn 0 cell of the player to the left of player 1? If not, can you adjust the formula to fix it?
- Does the formula in **G4** correctly refer to the turn 0 cell of the player to the right of player 10? If not, can you adjust the formula to fix it?

After you've checked and adjusted (if necessary) the formulas, try putting in data for turn 0, or the initial distribution of candy (cells **B3** through **G3** in this example). When you have some data for turn 0 in your spreadsheet, check the results of the formulas for turn 1:

- Do the results automatically update as you put in the turn 0 data?
- Do the formulas seem to be giving the correct values?
- How did you check the values? Are there any quick ways you can think of to check for major problems in the formulas?

Sharing candy for the remaining turns

As you've seen, when we copy and paste the sharing formulas, the type of cell references we're using (called *relative references*) automatically adjust to the destination cell. For example if a relative reference refers the cell above the current cell, and we copy and paste to another cell, the formula in the cell where we pasted will refer to the cell about that cell.

Spreadsheet programs provide many different features for copying the contents of one cell – or several cells – to an entire range of cells. Some of these involve selecting the cells to be copied, and then clicking and dragging a small box on the corner or edge of the outline of the selected cells. Others involve selecting a rectangular area of cells, with the cells that already containing the formulas of interest located at one edge of the selected area, and then using a fill function (sometimes found under the **Insert, Data, or Sheet** menu) to paste the formulas across the entire selection in a single operation.

Can you think of way to take advantage of relative cell addressing and copy/paste/fill features to apply the sharing formulas to the remaining turns in the game? What happens when you copy one (or all) of the turn 1 formulas to the corresponding cells in turn 2, turn 3, etc.? Try different ways of doing this, to see which works best for you, and which seems to make the most sense.

Fill in the sharing formulas for enough turns that when you experiment with different initial distributions of candy (turn 0 data), any pattern becomes apparent in the number of turns computed. (See Figure 2 for an example with 15 turns.)

	A	B	C	D	E	F	G
1		Players					
2	Turns	1	2	3	4	5	6
3	0	2	2	0	3	3	1
4	1	1	1	2	2	2	3
5	2	2	2	1	2	2	2
6	3	2	1	3	1	2	2
7	4	1	3	1	3	1	2
8	5	3	1	3	1	3	0
9	6	1	3	1	3	1	2
10	7	3	1	3	1	3	0
11	8	1	3	1	3	1	2
12	9	3	1	3	1	3	0
13	10	1	3	1	3	1	2
14	11	3	1	3	1	3	0
15	12	1	3	1	3	1	2
16	13	3	1	3	1	3	0
17	14	1	3	1	3	1	2
18	15	3	1	3	1	3	0

Figure 2: Candy sharing table with formulas copied for 15 turns.

Creating a graph

By now, you've probably seen a few different final patterns in the candy sharing data, with different initial distributions. How difficult was it to spot these patterns?

When you did the math circle candy sharing activity, did you try plotting the data for a game? What were the axes of your graph?

One useful graph for viewing the candy sharing game is a *time series* plot. This is a graph where time (or in this case, turns) appears on the *X* axis, and some changing quantity (such as the amount of candy) is shown on the *Y* axis. We can plot the candy time series for a single player by marking dots on the graph for the amount of candy that player has in her pile at each turn, and connecting those dots with line segments. We can even show candy for multiple players by using different colors (for example) for each player.

Spreadsheets make it easy to create time series plots, but there are some tricky details to watch out for:

- Most spreadsheet programs aren't very good at guessing the type of chart we want. So we have to be careful to choose one that works well for a time-series plot. In Microsoft Excel, LibreOffice Calc, and OpenOffice Calc, the best choice is an **XY (scatter)** chart, with connecting lines (and sometimes just lines displayed, without dot symbols); in Google Sheets, the best choice is a **Line** chart; in Apple Numbers, **2D Line** is the best option.
- The data for a single connected set of points is called a *series*. In this case, we probably want that each player to be a series. However, spreadsheet programs can sometimes guess wrong on this; for example, they might assume that each turn is a series, instead of each player. So when we create a time-series plot for candy sharing, we need to configure the chart accordingly: If each column in our table is a player, we need to specify that the data series of our plot are in columns, instead of rows.
- Since the *X* axis represents turns, we should generally include the turn numbers in the data that we select for plotting – but we have to make sure that the spreadsheet program understand that those are the *X* values of our plots, and not a separate series to be plotted.

For example, if we select the range **A2:G18** in Figure 2, and create an **XY (scatter)** chart in LibreOffice Calc, with connecting lines and no dot symbols, with data series in columns, and labels in the first row and first column, we will get a chart something like Figure 3. In Google Sheets, creating a **Line** chart with the same data gives us something like Figure 4.

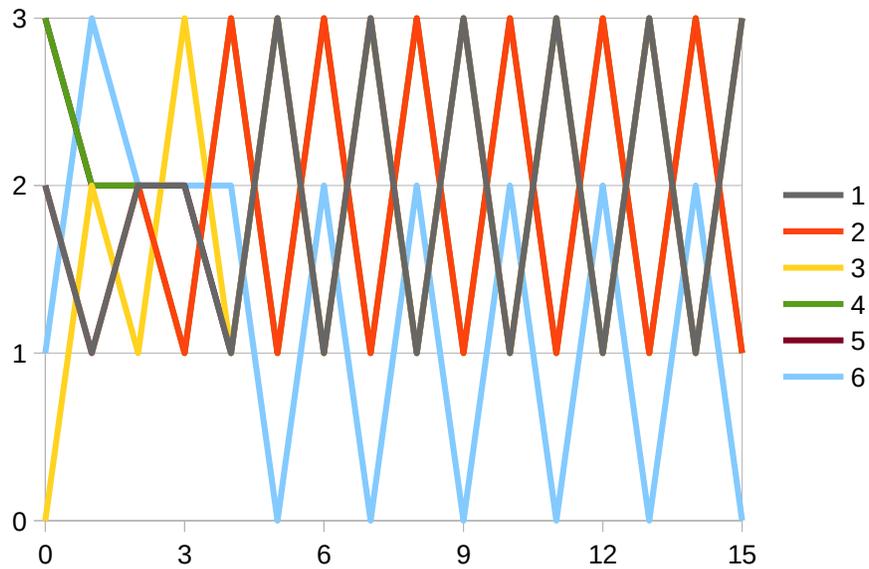


Figure 3: Candy sharing time series in LibreOffice Calc.

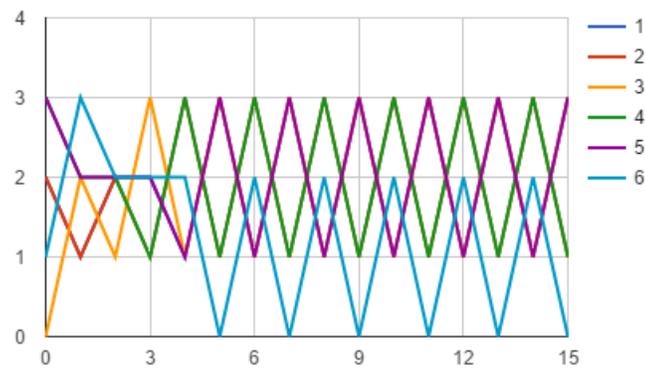


Figure 4: Candy sharing time series in Google Sheets.

Create a time series plot chart for your candy sharing table. Experiment with different chart options to find a combination that you think works well to present the results of that game clearly.

Challenges from math circle activity

Use your candy sharing table and time series plot to investigate some of the challenges and questions in the handouts for the candy sharing math circle activity. Are you able to make more progress towards solutions or answers, using the spreadsheet you've built?

Different number of players

Now that we've built a candy sharing table, with formulas that are doing the computation for us, and with an accompanying time series plot to make it easier to identify fixed points and cycles, how can we now adjust this approach for a game with more or fewer players?

One spreadsheet feature that can make this easier (or at least, safer) is the ability to copy an entire sheet (tab) to a new sheet. What do you think will happen with your formulas (specifically, with the cell references in those formulas) if you copy the entire sheet containing your current table to a new sheet?

See if you can find and try out the command to copy the entire sheet in this fashion. Sometimes the necessary command can be found when right-clicking (*Ctrl*-click on OS X) or clicking on a menu on the tab for the current sheet at the bottom of the spreadsheet window; in some cases, there is a **Copy Sheet** command under a **Sheet** menu at the top of the spreadsheet.

After copying the sheet, modify your table in the new sheet, to have a different number of players in the game. Do your formulas need to be adjusted? If you've added players, do you also need to add more turns (with the formulas copied appropriately) to the table?

When you have the formulas adjusted, don't forget about the time series plot; it will have to be adjusted as well – for a different number of players, as well as a different number of turns. Spreadsheet programs will do some of this adjustment automatically (for example, if you delete rows or columns, rather than simply clear the data from them, some spreadsheets will automatically remove corresponding series or data points from a chart); additionally, you can usually edit the data series of a chart, to add or remove points from them. In some case, however, you may find that after changing the dimensions of a table containing data used in a chart, the easiest way to adjust the chart to reflect those changes is to delete and recreate it.

After you've added and updated at least one new sheet, containing a candy sharing table for a different number of players, and a time series plot that correctly displays the new data, try it out: Investigate a few different initial distributions of candy in the new sheet. Does it seem to take more or fewer turns for the final pattern to become apparent, vs. the number of turns needed in your first table?