

The Game of Nim

The game of Nim consists of several piles of counters. Players alternate moves: select a pile, and remove at least one counter from that pile. The last player with a legal move wins.

The game of Jim consists of several rows of solid and empty disks. Play is similar: players alternate moves: select a row, and change one or more disks (solid to empty or empty to solid). The first disk to be changed must be solid (from the left, not necessarily the leftmost). The last player with a legal move wins.

Nim Challenges

Who has the winning strategy for each of the following games? The Next player or the Player?

1. N_0 (the empty Nim game with zero counters).
2. N_1, N_2, \dots, N_n (where n is a positive integer).
3. $N_1 + N_1$ (two Nim piles of one counter). $N_n + N_n$ (two identical Nim piles of $n > 0$ counters).
4. $N_3 + N_5$ (two Nim piles, 3 and 5 counters). $N_n + N_m$ (two different Nim piles, $m \neq n$.)
5. Make a conjecture about who has the winning strategy for two pile Nim games.
6. $N_3 + N_5 + N_8$ (three Nim piles, with 3, 5 and 8 counters).

Jim Challenges

Who has the winning strategy for each of the following games? The Next player or the Player?

1. One row Jim: $\circ \circ \circ$ (all empty disks). One row Jim: $\bullet \circ \bullet$.
2. What is the maximum number of moves in the one row Jim game $\bullet \circ \circ$?
3. Two row Jim: $\bullet \circ \bullet \mid \bullet \circ \bullet$. Two row Jim: $\circ \circ \bullet \mid \bullet \circ \bullet$.
4. Make a conjecture about who has the winning strategy for two row Jim games.
5. Three row Jim: $\circ \circ \bullet \bullet \mid \circ \bullet \circ \bullet \mid \bullet \circ \circ \circ$.

Nim and Jim

1. How long is the maximum number of moves in N_{17} (one Nim pile with 17 counters)?
2. Find a one row Jim game that has the same maximum number of moves as N_{17} .
3. Make a conjecture about the connection between Nim and Jim games.

For further reading

Elwyn R. Berlekamp John H. Conway and Richard K. Guy, *Winning Ways for your mathematical plays*, Academic Press, London, 1982.

Charles L. Bouton, *Nim, A Game with a Complete Mathematical Theory*, The Annals of Mathematics Vol. 3, No. 1/4 (1901 to 1902), 35–39.