## IN SESSION

## Liar's Bingo by Bob Klein and Steve Phelps

Patterns are one of Math Circles' great levelers. From recognizing a pattern to generating terms, to abstracting and making inferences, tasks based on patterns embody the "low-threshold, high-ceiling" trait of good problems. Liar's Bingo is all about patterns, and we have used it with kids aged 11 to 75 years old. We've had a Grade 3 special education teacher work side-by-side with an AP Calculus teacher to understand what makes it work, with each teacher finding challenges and rewards in the problem.

Liar's Bingo is played with strips of paper containing a 1 x 6 array of positive integers, some colored black and some red. Six sample strips are included in the figure at right (one strip per row). A complete set may be found at <a href="http://mathteacherscircle.org">http://mathteacherscircle.org</a>; these should be cut into individual strips and shuffled before using. Begin by passing out handfuls of strips to groups of participants. Ask them to "Put the strips in order." This request often leaves participants dumbfounded for a moment: "What order should we put them in?" or "Do you mean numerical order?" are typical questions. We are purposefully vague at this point, for we want the participants to study the cards and look for underlying patterns themselves.

Have the participants start working in pairs so they can "huddle" around the strips, and follow this with table discussions of 4-6 participants. Then, with the whole group, record found patterns on a chalkboard or chart paper for all to see. Take a moment to note what patterns you find in the figure at above right.

We have led this activity for groups with wideranging prior experience and have been surprised each time to have participants discover patterns that are new to us. Typically, groups will offer ideas related to the pattern of reds versus blacks, parity and order (increasing versus decreasing, for instance), placevalue, and maximum and minimum values. The better

41	21	11	5	3	0
17	77	47	53	<b>55</b>	56
31	<b>51</b>	61	75	73	70
16	76	46	<b>52</b>	54	57
55	35	5	11	17	14
34	54	64	70	76	75

sessions we've run have resulted from following Joshua Zucker's suggestion to "be less helpful"—participants get a long way when encouraged to struggle.

After charting a list of patterns, the facilitator announces that it is time to play Liar's Bingo. The rules of the game are simple: the facilitator will ask a participant to read (left-to-right) the sequence of six colors but to lie about exactly one of those colors. For instance, in the case of the first strip shown in the figure above, a participant might decide to lie about the fourth entry and say "black-black-black-red-black-red." The facilitator, without looking at the participant's strip, will divine that the participant has lied about the number 5 on the strip. The announcer will confirm that the facilitator was correct, to the amazement of the room. People generally ask to test the facilitator on another five or six strips.

The job of the room then becomes to figure out how the "trick" works. The facilitator becomes a black-box tester for sequences suggested by the participants. Some questions that help groups get unstuck include: "Independent of the strips in front of you now, what would be an interesting sequence of colors to ask about?" or "What sequence of colors would get me

to say 54? How about 45?" Eventually, people want to know about all-red or all-black strips, strips with all black except the rightmost entry, etc. Prospective facilitators are advised to let the groups struggle despite the strong desire to push them toward the answer. In our experience, the room gets there and the activity has a natural flow for about 1.25 hours.

Once people at a table "get the trick," their role shifts to first being "tested" by the table to see if they are able to replicate the trick and then to helping others discover the trick. One of the authors, at this point, finds it useful to wander the room announcing, "No spoilers! Your job is to help others discover for themselves."

To facilitate that same joy of discovery here, we offer the following "key" values based on playing the game with the strips in the figure (with one number's color lied about on each strip):

B-B-B-B-B	0
R-R-R-R-R	77
B-R-R-B-B-R	31
R-B-B-R-R-B	46
B-B-R-B-B-R	11
R-R-B-B-B	70

Spoiler alert! Before reading on, you may wish to pause to contemplate the explanation for the trick.

The underlying mathematics isn't that difficult and involves a mixture of decimal and binary representations. The first six entries on the strip represent the tens-digit of the number being lied about, but represented in binary. Hence the string beginning "B-R-R" would be 011, or 2 + 1 = 3. The last three entries on the strip represent the one's-digit being lied about, so "B-B-R" would be 001 or 1. This explains why the strip B-R-R-B-B-R above is coded "31."

Some great questions to explore at this point: Can you reverse-engineer a strip? Could you create your own set? How many strips could be constructed in

which "31" is the number being lied about? How many strips make a complete set? What would happen if someone lied about two colors? Are variants possible using eight entries per strip? Other bases?

"Closing" the activity is important because many of the participants will now be so focused on the "trick" that they will lose sight of the pattern recognition work at the beginning. Have the room revisit the patterns charted at the front of the room in the context of their new understanding of Liar's Bingo. This will easily push the session to two hours or more and may have to be assigned as "take-home" play.

Liar's Bingo is accessible to a wide audience and has, in our experience, hit that narrow band of "perplexity" or, to use another Circles' excellent phrase, "funstration," that gets under participants' skin yet needles them to persist to understand the problem. The "magic trick" performance is an important and memorable feature of the activity, and its occurrence near the middle of the session recharges the group's motivation. While teachers find this to be great fun, students love it, too, and sets of Liar's Bingo cards have been seen circulating school hallways in Puerto Rico, Thailand, Ohio, and the Navajo Nation.

Bob Klein teaches at Ohio University and is cofounder of the South East Ohio MTC. Steve Phelps teaches at Madeira High School and is co-founder of the Cincinnati Math Teachers' Circle. Phelps was first introduced to Liars' Bingo in a presentation by Chuck Sonenshine in a 1990 methods course. When asked for a copy of his cards, he suggested that Steve make his own, which he did that evening. You can find out more about "Mathemagician" Chuck Sonenshine on his website at <a href="http://www.keynotemath.com/">http://www.keynotemath.com/</a>.

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