*Math Teachers' Circle of Santa Fe*
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# *Sprouting Topologies*

Brussels Sprouts is a paper-and-pencil game where two players connect points (vertices) with lines (edges) according to simple rules. It is also about partitioning space. It leads to some interesting explorations…

First, the how to play:

1. One player draws some Xs (seeds) on the paper. I recommend three or four (no more!) for your first game. Note that Xs have four "tips".
2. One of the two players connects any two tips with a line (curved is fine). Tips can only be used once, so those two cannot be reused.
3. In order not to use up the tips, that player draws a short line through the midpoint of the just-drawn line, thus "restoring" the lost tips.
4. Without crossing any existing lines or Xs, the next player connects any other two open tips.
5. The last player able to draw a line wins.

Explorations:

1. Play some games.
2. What role does strategy play in Brussels Sprouts?
3. Is it possible to control the game so that you always win? How would you investigate this? What can you control about a game?
4. How long will a game last? You might restate this, "the number of moves in a game of Brussels Sprouts is a function of…"
5. Can you predict the number of total intersections (both the original Xs and the crosses you drew across the lines) that will exist at the end of a game? The minimum and maximum possible moves in a game?
6. There are three features (for you to count[1]) in a finished game of Brussels Sprouts:
    a. *Vertices*, or what we call each of the original Xs and each of the short lines you drew across the midpoints of the lines (the number "V").
    b. *Edges*, or the player-drawn lines ***between vertices*** (the number "E"). The "arms" of an X are not edges. When you draw a connecting line—an edge—you actually create *two new edges* when you draw the short line through midpoint of that connecting line.
    c. *Regions*, or the open areas enclosed by lines. This includes the region *outside* the drawn figure (the number "R").

    Can you determine a relationship between V, E, and R? How does this change between games?
7. Take your finished games and four different colored pens. Can you always find a way to color the vertices without ever coloring adjacent vertices the same color?
8. Some options (pick one):
    a. Imagine that you had played a game on the surface of a sphere. Would it have mattered to the course of the game?
    b. What if you had played Brussels Sprouts on a different surface—one of those listed below? Would you have employed a different strategy for each of them? Would the relationship between V, E, and R have changed?
        i. Cylinder
        ii. Möbius strip (we have adding machine paper and tape)
        iii. Torus
        iv. Two-holed torus
9. If you woke up one day and found you were somehow living on one of these surfaces, could playing Brussels Sprouts permit you to determine which one?
10. Change the rules:
    a. What if each vertex was a point that could accept four edges, but they were not limited to being connected on one side or another they way the tips above must be connected?
    b. What if each vertex was a point that could only accept three edges (the game of *Sprouts*)?

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[1] Counting suggestion: Number the *regions* in the centers, circle the *vertices* to make them easier to see and count, and write numbers directly on the *edges*.

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