Julia Robinson Mathematics Festival

Wolves and Sheep

Place 5 wolves on a 5 by 5 chessboard so that 3 sheep are safe. Wolves move like chess queens (any number of squares horizontally, vertically, or diagonally). A sheep is safe if no wolf can reach it in one turn.

For instance: the first grid below shows a solution with 6 wolves and room for 2 sheep. The squares marked W are wolves, x marks the unsafe spaces, and S marks the two safe sheep spaces.

W	x	W	W	x												
W	x	W	x	x												
x	x	x	x	W												
x	S	x	x	x												
x	S	x	x	x												

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Questions

1) What is the largest number of squares that 1 wolf can attack? What is the smallest number?
2) Solve a simpler problem: What is the largest number of sheep that can fit with 0 wolves? 1 wolf? 2 wolves?
3) What is the smallest number of wolves that can leave room for no safe sheep at all?
4) What is the smallest number of sheep that can fit with 0 wolves? 1 wolf? 2 wolves?

Harder Questions

5) How many different ways are there to place the 5 wolves on the board? (ignoring the sheep)
6) How many different ways are there to place the 3 sheep on the board? (ignoring the wolves)
7) How many different solutions are there to the original problem? Can you prove your answer?
8) Generalize: How many sheep can fit with w wolves on an n by n board? Perhaps some special cases (small values of w and/or n) would be easier to answer to help get you started. The 0 wolf and 1 wolf cases are probably a great starting point.