What is a Math Circle?

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To quote from the National Association of Math Circles site:

> *"Math Circles bring K-12 students or K-12 mathematics teachers together with mathematically sophisticated leaders in an informal setting, after school or on weekends, to work on interesting problems or topics in mathematics. The Circles combine significant content with a setting that encourages a sense of discovery and excitement about mathematics through problem solving and interactive exploration. Ideal problems are low-threshold, high-ceiling; they offer a variety of entry points and can be approached with minimal mathematical background, but lead to deep mathematical concepts and can be connected to advanced mathematics."*

Math circles can be run at any level—from K-12 to adults (teachers, mathematicians, parents, etc.). This does not mean that the problems are easy, even for younger students. They are certainly *not*. In general, problems will start from an easily grasped place—perhaps involving some physical object or manipulative—and build to a more generalized understanding of the area of mathematics involved.

For example, we might begin with a pile of candy, and distribute it in some way among five students (conference participants) at a table. Then we would have them share the candy in some regular pattern several different ways, with each new way helping them to discover something about the significance of the number of candies and candy sharers across the system. Or perhaps tables of students will be give strips of paper with numbers on them, then be told something as vague as "organize them." The circle leader will do his or her best to not be too helpful.

This math circle process is about discovery and invention—constructing mathematics from our innate (and teachable) ability to perceive patterns.

Mathematical circles are deeply about persevering through difficult problems that are often unlike any the student has seen before (Common Core Math Standard #1). Circle problems and session seldom start in any familiar place, and may seem to make little sense at first—but are highly attractive mysteries. Students must learn the standard "tools of the trade" of problem solving, such as working backwards, finding invariants, drawing pictures, making tables, the extreme principal, making a simpler problem/scale down, wishful thinking, and looking for symmetry. Further, since the leader will almost never reveal the answers, the students must learn to explain clearly their solutions and approaches—the beginnings of mathematical proof. Computing some quantity or following some rote algorithm will never be enough.

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[1] James Taylor runs the Math Circles Collaborative of New Mexico and Math Teachers' Circle of Santa Fe, in Santa Fe, New Mexico. He retired after 21 years at Santa Fe Preparatory School as Computer Department Head, computer and mathematics teacher, as well as Director of Technology. He has been working with math circles for students and teachers since an early 2005 tour of math circles in the San Francisco bay area, and has had an interest in provocative and subversive mathematics education much longer. James has taught many years of courses in mathematical problem solving at both the middle and high school levels, and has mentored teachers in circle approaches in the upper elementary school grades. His work is supported by the American Institute of Mathematics, the National Association of Math Circles, the Mathematical Association of America, and the Santa Fe Preparatory School. He has also been involved since the late 1990s in teaching computational science and computer modeling in the US and Mexico.

A math circle could not be more different than a traditional classroom, though they may follow many different styles and approaches. All participants must struggle through the problems, usually in small groups. The problems may range from open-ended explorations of a mathematical idea such as infinity or the nature of number, to math competition preparation, to the investigation of a game with important mathematical underpinnings (such as the card game *SET*[2]).

A math *teachers'* circle allows teachers to directly experience what students experience in a math circle. Further, the teachers get to have circle leading and pedagogy modeled for them by an experience leader—often a mathematician—and at some point get to try their hand at designing an leading a teacher circle. This experience will ideally lead to their introducing math circles at their schools to their students. The director of the Math Circles Collaborative of New Mexico and the Math Teacher's Circle of Santa Fe, James Taylor, is available to visit schools in the region and model circle activities in a teacher's classroom. There are math circles as well at Santa Fe Indian School, in Las Vegas, the Española Valley, Farmington, Los Lunas, and throughout the Navajo Nation.

There is a constellation of activities and events which includes student and teacher math circles, and extends the reach of the math circle approach: Math Wrangles and Julia Robinson Math Festivals.

A Math Wrangle (www.sanjosemathcircle.org/math-wrangle, with the rules at mathcircle.tamu.edu/wp-content/uploads/2014/09/math_wrangle_rules.pdf) is a *mathematics debate*. This is not as strange as it sounds. Teams of students go off into a room to study a set of eight problems, then return to the debate setting and the coin toss. The winning toss permits the team to challenge the opponents to solve one of the problems. That team may accept, and a member presents their solution, deals with any rebuttal to their solution, and judges award points. There is more to it, but in my experience students *love* the experience. Mounting a team requires training, as does any debate program.

A math festival is a terrific way to introduce large numbers of students, teachers, and community members to how playful and fun mathematics can be, math circle-style. The first New Mexico Julia Robinson Math Circle (jrmf.org) was held February 24, 2017 at Santa Fe Community College (stemsantafe.org/news-events/julia-robinson-mathematics-festival-santa-fe/) with about 150 middle school students and about 50 adult activity-table leaders. Students get to freely choose math activities and play with them for about an hour-and-a-half. A festival engages not only students, but also schools, teachers, the adult table leaders (volunteers), along with alerting the larger community to some positive buzz about mathematics.

There are sample math circle videos available on a YouTube channel at youtube.com/user/MathCircles/videos.

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[2] For starters, SET can be used to introduce isomorphism, topology/torus vs. plane/finite unbounded vs. finite bounded, magic squares, unordered sets, dimension, finite discrete dimensions, combinatorics, modular arithmetic, projective geometry, multi-dimensional geometry, error-correcting codes, non-geometric dimension, probability, statistics, modeling, linear algebra/vectors, number theory, caps sets, seeing patterns, and more.