Table Leader Guide

Julia Robinson Mathematics Festival
Santa Fe Community College
Friday February 24, 2017

JRMF table leader orientation

1. Intro
   a. welcome and thank you
   b. instructors & table leaders introduce selves
   c. intergalactic math salute
   d. team formation and tub distribution (e.g., (What is a Math Circle")
   e. math circle pledge
2. Overview of the festival (the first ever NM JRMF!!!)
3. **Investigation of your table activities!**
4. How to be a JRMF table leader
5. Review of the Festival schedule
6. Final Q & A
7. Quick look at supplemental table leader support documents
8. Photo release form to fill out and sign

Overview
You will be "leading" a table—one of two adjacent tables running the same math activity. There will be from two to four of you at each of those pairs of tables. Students will circulate throughout the room until they find an activity that interests them. Feel free to tout your activity!

The activities are designed to be easy to start (low threshold), but most of them get very deep (high ceiling)—hard enough that the activity designers don't know the answers to the final questions.

Your own investigation
We will give your activity team a tub containing your activity, instructions, and materials/manipulatives. Dive in! If something is unclear to you, then ask away! We may or may not help you at first. We do need to know if something about the instructions is unclear or if there are *missing* materials/manipulatives. And…**no devices** (for you or the students at your table)! There is nothing here that requires them, and they are counterproductive.

Can you imagine doing this—your table activity—with a constantly changing group of 7th and 8th graders for two hours?

Did anyone dominate your group? How would you manage this with the students are your table?

How to be a JRMF table leader

The dance of the JRMF table leader

If you are a teacher at any level you may find the role of table leader at a JRMF table (or as a math circle leader someday!) a bit of a change of pace:

Your role is not to teach anything at all!

Whoa. Even though you will have explored the activity in your training session, you will likely feel unprepared to teach it. So no worries. *That is your ideal condition*.

A big part of your job is to help students both stay engaged and to *have a good time*. Our overall goal is to keep students "productively stuck" on the activity, rather than to get them through the activity and to whatever answers they can come up with as quickly as possible. If they are engaged, you have very little to do—a good thing. If they are stuck or frustrated, your job is to get them moving again without giving away too much. And to help them see the puzzle as a chance to play. Remind them that problem solving need not be a solitary game—enlist your tablemates. Suggestions for how to do all this are in the next section.

Your motto: **Be less helpful**

During the festival, should your table be well-covered by your team, feel free to take a break and visit other activity tables. If it seems appropriate, consider asking the students to explain what their activity is and what they have discovered.

Asking students good mathematical questions

The Julia Robinson Math Festival session leader Guide provides some great points about this...

- *The kind of communication you do depends a lot on the kid. For some, you'll be helping them read and understand the questions.*
- *For a more advanced student, you'll be reminding them of key strategies like*
    - *Can you try a simpler example?*
    - *Can you do a related problem?*
    - *Draw a picture*
- *Maybe they already solved the problem, and you want to guide them toward a different way of looking at it. Generally, try to be the one asking questions more than answering them.*
    - *Can you explain that to me?*
    - *Why does that have to be so?*
    - *Can you show me an example?*
- *It is also good to be prepared with some questions to ask when kids make a mistake or get something completely wrong.*
    - *Try to say "Why?" with the same tone whether they have it right or wrong -- make them explain it to you and uncover their own error.*
    - *Talk about ways they can check their answer, again whether it is right or wrong.*

Julia Robinson Mathematics Festival

- *If you hear another kid putting someone down for not knowing an answer or for getting something wrong, be prepared with a remark that suits you. I like to remind them that we're here to explore and learn, and it would be boring if we already knew how to do all the problems.*

Teach students how to ask good mathematical questions

From Sam Vandervelde's *Circle in a Box:*

Students almost never have the opportunity to ask, "What if . . . ?" types of questions at any point during their secondary school careers. They have no idea that one of the most crucial skills acquired by a professional mathematician is the ability to ask productive questions; the sorts of questions that lead to new areas of research. All of their training suggests that mathematics is synonymous with solving problems; very few of them stop to wonder where the problems come from. Moreover, working on problems can become tiresome or frustrating. But contemplating new directions to explore, free from the burden of needing to answer all the questions that might arise (at least for the time being), is a marvelous, creative endeavor. Every student should be given the chance to practice this process.

So take a few minutes after wrapping up a nice problem to point out that the book is not yet closed on this particular idea, and ask students where it might lead. At first students may need a lot of coaxing. What happens if one uses different numbers or shapes? Is there an analogous result in higher dimensions? What if we allowed three people to play this game, how would that look? Encourage any ideas or attempts; quite often once the first question or two is tentatively offered the floodgates are opened. Help students refine vague ideas into well formulated questions. This activity can be as rewarding for the leader as for the students— it is exciting to see what they come up with, and invariably everyone leaves with new ideas to pursue.

And again…

Don't try to teach! Enjoy seeing how quickly young minds play with new activities and go off in all sorts of crazy directions. As adults, we often try to channel these wild speculations, but who knows where they may lead an inquisitive young mathematician. I have often found middle schoolers to be faster at gaining insights or in finding surprising lateral directions to explore than older kids or adults.

JRMF Agenda

8:00-8:30	Volunteers Check-in and Breakfast
8:30-9:30	Table leader program led by Josh Zucker
9:30-10:00	Teachers/Students check-in, Table Leaders set up activities
10:00-12:00	Math Festival– Jemez Rooms
10:00-12:00	Math Circle for Teachers and Chaperones led by Josh Zucker
12:00-12:30	Surveys and Raffle led by SFCC President
12:30-1:30	Lunch and Group Activity led by Josh Zucker

What is a Math Circle?

by James Taylor jtaylor@sfprep.org

Math Circles Collaborative of New Mexico
Math Teachers' Circle of Santa Fe

To quote from the National Association of Math Circles site:

> *"Math Circles bring K-12 students or K-12 mathematics teachers together with mathematically sophisticated leaders in an informal setting, after school or on weekends, to work on interesting problems or topics in mathematics. The Circles combine significant content with a setting that encourages a sense of discovery and excitement about mathematics through problem solving and interactive exploration. Ideal problems are low-threshold, high-ceiling; they offer a variety of entry points and can be approached with minimal mathematical background, but lead to deep mathematical concepts and can be connected to advanced mathematics."*

Math circles can be run at any level—from K-12 to adults (teachers, mathematicians, parents, etc.). This does not mean that the problems are easy, even for younger students. They are certainly *not*. In general, problems will start from an easily grasped place—perhaps involving some physical object or manipulative—and build to a more generalized understanding of the area of mathematics involved.

For example, we might begin with a pile of candy, and distribute it in some way among five students (conference participants) at a table. Then we would have them share the candy in some regular pattern several different ways, with each new way helping them to discover something about the significance of the number of candies and candy sharers across the system. Or perhaps tables of students will be give strips of paper with numbers on them, then be told something as vague as "organize them." The circle leader will do his or her best to not be too helpful.

This math circle process is about discovery and invention—constructing mathematics from our innate (and teachable) ability to perceive patterns.

Mathematical circles are deeply about persevering through difficult problems that are often unlike any the student has seen before (Common Core Math Standard #1). Circle problems and session seldom start in any familiar place, and may seem to make little sense at first—but are highly attractive mysteries. Students must learn the standard "tools of the trade" of problem solving, such as working backwards, finding invariants, drawing pictures, making tables, the extreme principal, making a simpler problem/scale down, wishful thinking, and looking for symmetry. Further, since the leader will almost never reveal the answers, the students must learn to explain clearly their solutions and approaches—the beginnings of mathematical proof. Computing some quantity or following some rote algorithm will never be enough.

A math circle could not be more different than a traditional classroom, though they may follow many different styles and approaches. All participants must struggle through the problems, usually in small groups. The problems may range from open-ended explorations of a mathematical idea such as infinity or the nature of number, to math competition preparation, to the investigation of a game with important mathematical underpinnings (such as *Set*).

A math *teachers'* circle allows teachers to directly experience what students experience in a math circle. Further, the teachers get to have circle leading and pedagogy modeled for them by an experience leader—often a mathematician—and at some point get to try their hand at designing an leading a teacher circle. This experience will ideally lead to their introducing math circles at their schools to their students.

Finally, bit of math circle history[1]

Math circles originated in Hungary more than a century ago. They soon spread over Eastern Europe and Asia, and since then have produced many of the great scientists from those parts of the world, in mathematics and in other disciplines. The math circles also led eventually to the start of many national and international math contests, including the International Mathematical Olympiad (IMO) in 1959 in Romania. It is widely believed that it is the presence of these circles that has enabled the youth of countries such as Russia, Bulgaria and Romania on average to outperform the United States at the IMO. Given the success of math circles in Russia and Eastern Europe, it is surprising that it has taken so long for the United States to develop similar programs.

The Berkeley Math Circle was founded in 1998 to begin to correct this situation. Since then groups have begun in Palo Alto and San Jose. The San Francisco Bay Area was a natural choice for the site of math circles, both because of the large number of talented high school students in the area, and because of the proximity of world-class institutions such as MSRI from which experienced lecturers could be drawn.

[1] Again from the National Association of Math Circles, https://www.mathcircles.org/content/history-american-math-circles

Julia Robinson Mathematics Festival

STEM SANTA FE

Math Circle Problem-Solving Strategies

Compiled by Bay Area Math Circle session leader Emily McCullough, with assistance from Josh Zucker and Tom Davis. http://bact.mathcircles.org/files/Summer2013/Problem%20Solving%20Strategies.pdf

1. **Do something. Get your hands dirty. Play.**
2. **Patience.**
   Remember the difference between a problem and an exercise. *This is a problem.*
3. **Start small and simple.**
   Build from there.it is almost always useful to look at the very smallest versions of a problem. They are a lot easier to work out, usually.
4. **Special and Specific Cases.**
   - Look at specific cases or particular configurations.
   - Work out a specific example or play the game a few times. See what happens...
   - Look for extremes.
   - Make and solve an easier problem.
5. **Organization.**
   - If you are working on a problem that can be split into cases, make sure you have got all the cases.
   - If you are looking at examples, or doing experimental calculations, keep track of this information in an organized way.
   - Experiment with multiple ways of recording and presenting your findings. One picture or graph or table may be better than another for seeing patterns or visualizing your results.
6. **Look for a pattern.**
   Start small and simple here too. Patterns and formulas that hold for these specific cases often provide a clue for the general case.
7. **Generalize.**
   Sometimes generalizing makes a problem easier. Variables are not always scarier or harder to work with. And you can prove something more with less!
8. **Symmetry.**
9. **Wishful Thinking.**
   Factor $x^4 + x^2+1$. This seems difficult, but it would be easy if you had $2x^2$ instead of x^2. So just change it to what you want (but correct it too):
   $$x^4+x^2+1= x^4+2x^2-x^2+1= (x^2+1)^2 -x^2$$
   But that is just the difference of two squares, so:
   $$x^4 + x^2 + 1= (x^2+1-x)^2 (x^2+1+x)^2$$
10. **Work Backwards.**
11. **Recycle. (Reduce, Reuse, Recycle)**
    If you have worked out values for simpler versions of the problem, perhaps you can use them to work out harder versions.
12. **Give things names.**
    Give a name or symbol to new objects and operations you are using. Also name the mathematical properties and objects you use and observe while problem-solving (e.g. commutative property of addition, divisors, parity). This is a great opportunity to use student names to name theorems/ideas by the Math Circle group, ex: Sam's Theorem.
13. **Make a picture.**
14. **Invariants. Monovariants.**
    Sometimes you can find a quantity that is unchanged after every operation (called an invariant) and sometimes you can find a quantity that changes in one direction (called a monovariant).